


Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Date: 28 – 12 – 2014 Course: Mathematics 1 – A Duration: 3 hours
<ul style="list-style-type: none"> • Answer All questions • The Exam Consists of One page 	<ul style="list-style-type: none"> • No. of questions: 4 • Total Mark: 100 Marks 	
<p>[1] Find y' from the following:</p> <p>(a) $y = 3x^{-4} + 3^{\sin x} + \frac{1}{2}$ (b) $y = \cosh 2x \cdot \sinh x^2$ (c) $y = \tan x \cdot \ln(2^x + \sqrt{3})$</p> <p>(d) $y = \tanh^{-1}x^2 + \sin^{-1}x$ (e) $y = x^{\sqrt{x}} + (\sinh x)^x$ (f) $y \cos x + x \sin y = 2$</p> <p>(g) $y = \frac{\sqrt{\sin^5 x + \sin x^5}}{\sqrt[3]{x + \cos x} \cdot \sqrt[4]{x + \cosh x}}$ (h) $y = t^2 + \ln \cos t, \quad x = t + \tan^{-1} \ln t$</p>	24	
<p>[2](a) Find the following limits:</p> <p>(i) $\lim_{x \rightarrow 0} \frac{x^4 + \sin^4 x}{x^5 + \tan^4 x}$ (ii) $\lim_{x \rightarrow \infty} (x - \ln(1 + e^x))$ (iii) $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x)$</p> <p>(b) Determine the maximum and minimum points of the functions:</p> <p>$f(x) = x \cdot e^{-x}$, $g(x) = 1 - x^{\frac{2}{3}}$, $h(x) = x^2 - 4 \ln(x + 1)$.</p> <p>(c) Show that $\operatorname{sech}^{-1} x = \ln(1 + \sqrt{1 - x^2}) - \ln x$</p> <p>(d) Write the Maclurin's series of the function: $f(x) = x \ln(2x + 1)$</p>	9 9 4 4	
<p>[3](a) Find the integrals: (i) $\int \frac{x^2 dx}{(4 - x^2)^{3/2}}$ (ii) $\int x^2 \tan^{-1} x dx$ (iii) $\int \cos^3 x \sin^{-4} x dx$</p> <p>(b) Find the area bounded by: $r = \frac{6}{1 + \cos \theta}$ and y-axis.</p> <p>(c) Find the circumference of cardioid: $r = a(1 - \cos \theta)$</p>	15 5 5	
<p>[4](a) Find the integrals: (i) $\int \frac{1}{x^{5/2} (3x + 1)^{3/2}} dx$ (ii) $\int \frac{\sin x dx}{\cos x (1 + \cos^2 x)}$</p> <p>(b) If $I_m = \int \tan^m x dx$ show that $I_m = \frac{\tan^{m-1} x}{m-1} - I_{m-2}$</p> <p>(c) Find the volume generated by revolving the cycloid: $x = \theta - \sin \theta, \quad y = 1 - \cos \theta$ about y – axis.</p> <p>(d) Find the area of the surface of revaluation generated by revolving the hypocycloid $x = a \cos^3 \theta, \quad y = a \sin^3 \theta$ about x-axis.</p>	10 5 5 5	

[1] Prove that: $\sinh^{-1} x = \ln(x + \sqrt{1 + x^2})$ and show that it is odd function.

[2] Find (a) $\lim_{x \rightarrow 0} \frac{\tan 2x}{1 - 2^{3x}}$

(b) $\lim_{x \rightarrow 2} \frac{\ln(x - 1)}{2 - x}$

(c) $\lim_{x \rightarrow \infty} \frac{x + 2^x}{x^2 - 3^{2x}}$

[3] Find y' where

(a) $y = 2x^3 - 3 \sin 2x$

(b) $y = \tan^{-1} x^2 + \tan^{-2} x$

(c) $y = 5^{x^3} - \tanh^{-1} x$

(d) $y = \cosh x^2 - \operatorname{sech} x$

(e) $y = \ln \cos x + (\ln x)^x$

(f) $y = \sin^{-1} 3^x \cdot \sinh^{-1} x^3$

(g) $y = \sin^5 x \cdot \sin x^5$

(h) $y = \ln \frac{\sqrt[4]{x + \cosh x} \cdot (x + \ln x)^4}{\sqrt[5]{x^2 + \cos 2x} \cdot (x - \tan x)^5}$